Algebra II

This discipline complements and expands the mathematical content and concepts of algebra I and geometry. Students who master algebra II will gain experience with algebraic solutions of problems in various content areas, including the solution of systems of quadratic equations, logarithmic and exponential functions, the binomial theorem, and the complex number system.

1.0   Students solve equations and inequalities involving absolute value.

2.0   Students solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs, or with matrices.

3.0   Students are adept at operations on polynomials, including long division.

4.0   Students factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes.

5.0   Students demonstrate knowledge of how real and complex numbers are related both arithmetically and graphically. In particular, they can plot complex numbers as points in the plane.

6.0   Students add, subtract, multiply, and divide complex numbers.

7.0   Students add, subtract, multiply, divide, reduce, and evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions, including those with negative exponents in the denominator.

8.0   Students solve and graph quadratic equations by factoring, completing the square, or using the quadratic formula. Students apply these techniques in solving word problems. They also solve quadratic equations in the complex number system.

9.0   Students demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is, students can determine how the graph of a parabola changes as *a, b,* and *c* vary in the equation $y = a(x\text{-}b)^2 + c$.

10.0   Students graph quadratic functions and determine the maxima, minima, and zeros of the function.

11.0   Students prove simple laws of logarithms.

   11.1   Students understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

   11.2   Students judge the validity of an argument according to whether the properties of real numbers, exponents, and logarithms have been applied correctly at each step.

12.0   Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay.

13.0   Students use the definition of logarithms to translate between logarithms in any base.

14.0   Students understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.

15.0   Students determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true.

16.0   Students demonstrate and explain how the geometry of the graph of a conic section (e.g., asymptotes, foci, eccentricity) depends on the coefficients of the quadratic equation representing it.

17.0 Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, students can use the method for completing the square to put the equation into standard form and can recognize whether the graph of the equation is a circle, ellipse, parabola, or hyperbola. Students can then graph the equation.

18.0 Students use fundamental counting principles to compute combinations and permutations.

19.0 Students use combinations and permutations to compute probabilities.

20.0 Students know the binomial theorem and use it to expand binomial expressions that are raised to positive integer powers.

21.0 Students apply the method of mathematical induction to prove general statements about the positive integers.

22.0 Students find the general term and the sums of arithmetic series and of both finite and infinite geometric series.

23.0 Students derive the summation formulas for arithmetic series and for both finite and infinite geometric series.

24.0 Students solve problems involving functional concepts, such as composition, defining the inverse function and performing arithmetic operations on functions.

25.0 Students use properties from number systems to justify steps in combining and simplifying functions.

Trigonometry

Trigonometry uses the techniques that students have previously learned from the study of algebra and geometry. The trigonometric functions studied are defined geometrically rather than in terms of algebraic equations. Facility with these functions as well as the ability to prove basic identities regarding them is especially important for students intending to study calculus, more advanced mathematics, physics and other sciences, and engineering in college.

1.0 Students understand the notion of angle and how to measure it, in both degrees and radians. They can convert between degrees and radians.

2.0 Students know the definition of sine and cosine as y- and x-coordinates of points on the unit circle and are familiar with the graphs of the sine and cosine functions.

3.0 Students know the identity $\cos^2 (x) + \sin^2 (x) = 1$:

3.1 Students prove that this identity is equivalent to the Pythagorean theorem (i.e., students can prove this identity by using the Pythagorean theorem and, conversely, they can prove the Pythagorean theorem as a consequence of this identity).

3.2 Students prove other trigonometric identities and simplify others by using the identity $\cos^2 (x) + \sin^2 (x) = 1$. For example, students use this identity to prove that $\sec^2 (x) = \tan^2 (x) + 1$.

4.0 Students graph functions of the form $f(t) = A \sin (Bt + C)$ or $f(t) = A \cos (Bt + C)$ and interpret A, B, and C in terms of amplitude, frequency, period, and phase shift.

5.0 Students know the definitions of the tangent and cotangent functions and can graph them.

6.0 Students know the definitions of the secant and cosecant functions and can graph them.

7.0 Students know that the tangent of the angle that a line makes with the x-axis is equal to the slope of the line.

8.0 Students know the definitions of the inverse trigonometric functions and can graph the functions.

9.0 Students compute, by hand, the values of the trigonometric functions and the inverse trigonometric functions at various standard points.

10.0 Students demonstrate an understanding of the addition formulas for sines and cosines and their proofs and can use those formulas to prove and/or simplify other trigonometric identities.

11.0 Students demonstrate an understanding of half-angle and double-angle formulas for sines and cosines and can use those formulas to prove and/or simplify other trigonometric identities.

12.0 Students use trigonometry to determine unknown sides or angles in right triangles.

13.0 Students know the law of sines and the law of cosines and apply those laws to solve problems.

14.0 Students determine the area of a triangle, given one angle and the two adjacent sides.

15.0 Students are familiar with polar coordinates. In particular, they can determine polar coordinates of a point given in rectangular coordinates and vice versa.

16.0 Students represent equations given in rectangular coordinates in terms of polar coordinates.

17.0 Students are familiar with complex numbers. They can represent a complex number in polar form and know how to multiply complex numbers in their polar form.

18.0 Students know DeMoivre's theorem and can give nth roots of a complex number given in polar form.

19.0 Students are adept at using trigonometry in a variety of applications and word problems.